IN THIS CHAPTER

- » Identifying the different types of numbers
- » Placing numbers on a number line
- » Becoming familiar with the vocabulary
- » Recognizing the operations of algebra

Chapter 1 Assembling Your Tools: Number Systems

You've undoubtedly heard the word *algebra* on many occasions, and you know that it has something to do with mathematics. Perhaps you remember that algebra has enough stuff in it to require taking two separate high school algebra classes — Algebra I and Algebra II. But what exactly is algebra? What is it really used for?

This book answers these questions and more, providing the straight scoop on some of the contributions to algebra's development, what it's good for, how algebra is used, and what tools you need to make it happen. In this chapter, you find some of the basics necessary to make it easier to find your way through the different topics in this book.

In a nutshell, *algebra* is a way of generalizing arithmetic. Through the use of *variables* (letters representing numbers) and formulas or equations involving those variables, you solve problems. The problems may be in terms of practical applications, or they may be puzzles for the pure pleasure of the solving. Algebra uses positive and negative numbers, integers, fractions, operations, and symbols to analyze the relationships between values. It's a systematic study of numbers and their relationships, and it uses specific rules.

Identifying Numbers by Name

Where would mathematics and algebra be without numbers? Numbers aren't just a part of everyday life, they are the basic building blocks of algebra. Numbers give you a value to work with. Where would civilization be today if not for numbers? Without numbers to figure the distances, slants, heights, and directions, the pyramids would never have been built. Without numbers to figure out navigational points, the Vikings would never have left Scandinavia. Without numbers to examine distance in space, humankind could not have landed on the moon.

Even the simplest tasks and the most common of circumstances require a knowledge of numbers. Suppose that you wanted to figure the amount of gasoline it takes to get from home to work and back each day. You need a number for the total miles between your home and business, and another number for the total miles your car can run on a gallon of gasoline.

The different sets of numbers are important because what they look like and how they behave can set the scene for particular situations or help to solve particular problems. It's sometimes really convenient to declare, "I'm only going to look at whole-number answers," because whole numbers don't include fractions or negatives. You could easily end up with a fraction if you're working through a problem that involves a number of cars or people. Who wants half a car or, heaven forbid, a third of a person?

Algebra uses different sets of numbers, in different circumstances. I describe the different types of numbers here.

Realizing real numbers

Real numbers are just what the name implies. In contrast to imaginary numbers, they represent *real* values — no pretend or make-believe. Real numbers cover the gamut and can take on any form — fractions or whole numbers, decimal numbers that can go on forever and ever without end, positives and negatives. The variations on the theme are endless.

Counting on natural numbers

A *natural number* (also called a *counting number*) is a number that comes naturally. What numbers did you first use? Remember someone asking, "How old are you?" You proudly held up four fingers and said, "Four!" The natural numbers are the numbers starting with 1 and going up by ones: 1, 2, 3, 4, 5, 6, 7, and so on into infinity. You'll find lots of counting numbers in Chapter 8, where I discuss prime numbers and factorizations.

Whittling out whole numbers

Whole numbers aren't a whole lot different from natural numbers. Whole numbers are just all the natural numbers plus a 0: 0, 1, 2, 3, 4, 5, and so on into infinity.

Whole numbers act like natural numbers and are used when whole amounts (no fractions) are required. Zero can also indicate none. Algebraic problems often require you to round the answer to the nearest whole number. This makes perfect sense when the problem involves people, cars, animals, houses, or anything that shouldn't be cut into pieces.

Integrating integers

Integers allow you to broaden your horizons a bit. Integers incorporate all the qualities of whole numbers and their opposites (called their *additive inverses*). *Integers* can be described as being positive and negative whole numbers and zero: −3, −2, −1, 0, 1, 2, 3.

Integers are popular in algebra. When you solve a long, complicated problem and come up with an integer, you can be joyous because your answer is probably right. After all, it's not a fraction! This doesn't mean that answers in algebra can't be fractions or decimals. It's just that most textbooks and reference books try to stick with nice answers to increase the comfort level and avoid confusion. This is my plan in this book, too. After all, who wants a messy answer — even though, in real life, that's more often the case. I use integers in Chapter 14 and those later on, where you find out how to solve equations.

Being reasonable: Rational numbers

Rational numbers act rationally! What does that mean? In this case, acting rationally means that the decimal equivalent of the rational number behaves. The decimal eventually ends somewhere, or it has a repeating pattern to it. That's what constitutes "behaving."

Some rational numbers have decimals that end such as: 3.4, 5.77623, −4.5. Other rational numbers have decimals that repeat the same pattern, such as $3.164164\overline{164}$, or $0.66666666\overline{6}$. The horizontal bar over the 164 and the 6 lets you know that these numbers repeat forever.

In *all* cases, rational numbers can be written as fractions. Each rational number has a fraction that it's equal to. So one definition of a *rational number* is any number that can be written as a fraction, $\frac{p}{q}$, where p and q are integers (except q can't be 0). If a number can't be written as a fraction, then it isn't a rational number. Rational numbers appear in Chapter 16, where you see quadratic equations, and later, when the applications are presented.

Restraining irrational numbers

Irrational numbers are just what you may expect from their name: the opposite of rational numbers. An *irrational number* cannot be written as a fraction, and decimal values for irrationals never end and never have a nice pattern to them. Whew! Talk about irrational! For example, π, with its never-ending decimal places, is irrational. Irrational numbers are often created when using the quadratic formula, as you see in Chapter 16, because you find the square roots of numbers that are not perfect squares, such as: $\sqrt{6}$ and $\sqrt{85}$.

Picking out primes and composites

A number is considered to be *prime* if it can be divided evenly only by 1 and by itself. The prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, and so on. The only prime number that's even is 2, the first prime number. Mathematicians have been studying prime numbers for centuries, and prime numbers have them stumped. No one has ever found a formula for producing all the primes. Mathematicians just assume that prime numbers go on forever.

A number is *composite* if it isn't prime — if it can be divided by at least one number other than 1 and itself. So the number 12 is composite because it's divisible by 1, 2, 3, 4, 6, and 12. Chapter 8 deals with primes, but you also see them throughout the chapters, where I show you how to factor primes out of expressions.

Numbers can be classified in more than one way, the same way that a person can be classified as male or female, tall or short, blonde or brunette, and so on. The number −3 is negative, it's an integer, it's an odd number, it's rational, and it's real. The number −3 is also a negative prime number. You should be familiar with all these classifications so that you can read mathematics correctly.

Zero: It's Complicated

Zero is a very special number. It wasn't really used in any of the earliest counting systems. In fact, there is no symbol for zero in the Roman numerals!

Zero is a very useful number, but it also comes with its challenges. You can't divide by zero, but you can add zero to a number and multiply a number by 0. You'll find zero popping up in the most interesting places!

Imagining imaginary numbers

Yes, there are imaginary numbers in mathematics. These numbers were actually created by mathematicians who didn't like not finishing a problem! They would be trying to solve a quadratic equation and be stumped by the situation where they needed the square root of a negative number. There was no way to deal with this.

So some clever mathematicians came up with a solution. They declared that $\sqrt{-1}$ must be equal to i. Yes, the i stands for "imaginary." You'll see how this works in Chapter 16.

Coping with complex numbers

A complex number isn't really all that mysterious. This is just a designation that allows for you to deal with both real and imaginary parts of a number. A complex number has some of each! Complex numbers have the general format of $a+bi$, where a and b are real numbers, and the i is that imaginary number, $\sqrt{-1}$.

EXAMPLE

Q. Using the choices: natural, whole, integer, rational, irrational, prime, and imaginary, which of these can be used to describe the number 8?

A. **Natural, whole, integer, rational.** The number 8 fits all of these descriptions. It is rational, because you can write it as a fraction such as $\frac{8}{1}$ or $\frac{24}{3}$.

Q. Using the choices: natural, whole, integer, rational, irrational, prime, and imaginary, which of these can be used to describe the number $-\frac{2}{3}$?

A. **Rational.** This is written as a fraction but cannot be reduced to create an integer.

Q. Using the choices: natural, whole, integer, rational, irrational, prime, and imaginary, which of these can be used to describe the number $\sqrt{17}$?

A. **Irrational.** The number 17 isn't a perfect square, so the decimal equivalence of $\sqrt{17}$ is a decimal that goes on forever without repeating or terminating.

Q. Using the choices: natural, whole, integer, rational, irrational, prime, and imaginary, which of these can be used to describe the number $\sqrt{-9}$?

A. **Imaginary.** Even though 9 is a perfect square, so you can write the number as $\sqrt{-1}\cdot\sqrt{9}$ and then simplify it to read $i \cdot 3$ or $3i$, this number stays imaginary.

YOUR TURN

1. Identify which of the following numbers are natural numbers:

$$-41,\ 15,\ -5.2,\ 11,\ 3.2121...,\ -\frac{12}{3},\ \frac{14}{11},\ \sqrt{-5},\ \sqrt{10},\ \sqrt{9}$$

2. Identify which of the following numbers are integers:

$$-41,\ 15,\ -5.2,\ 11,\ 3.2121...,\ -\frac{12}{3},\ \frac{14}{11},\ \sqrt{-5},\ \sqrt{10},\ \sqrt{9}$$

3. Identify which of the following numbers are rational numbers:

$$-41,\ 15,\ -5.2,\ 11,\ 3.2121...,\ -\frac{12}{3},\ \frac{14}{11},\ \sqrt{-5},\ \sqrt{10},\ \sqrt{9}$$

4. Identify which of the following numbers are irrational numbers:

$$-41,\ 15,\ -5.2,\ 11,\ 3.2121...,\ -\frac{12}{3},\ \frac{14}{11},\ \sqrt{-5},\ \sqrt{10},\ \sqrt{9}$$

5. Identify which of the following numbers are prime numbers:

$$-41,\ 15,\ -5.2,\ 11,\ 3.2121...,\ -\frac{12}{3},\ \frac{14}{11},\ \sqrt{-5},\ \sqrt{10},\ \sqrt{9}$$

6. Identify which of the following numbers are imaginary numbers:

$$-41,\ 15,\ -5.2,\ 11,\ 3.2121...,\ -\frac{12}{3},\ \frac{14}{11},\ \sqrt{-5},\ \sqrt{10},\ \sqrt{9}$$

Placing Numbers on the Number Line

A number line is labeled with numbers that increase as you move from left to right. And numbers are listed with an equal amount or value between any two consecutive numbers.

Numbers are placed on a number line to give you a visual picture of how they compare, how far apart they are, and what is missing between them. The two number lines shown here are examples of some versions that are possible. In Figure 1-1, you see the half-way mark indicated between units. And in Figure 1-2, the negative and positive integers are shown, with 0 in the middle.

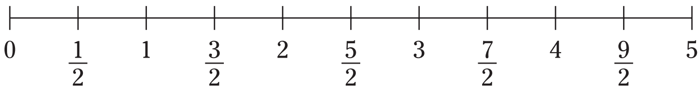


FIGURE 1-1: A number line from 0 to 5 with half-unit increments.

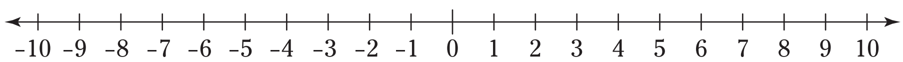
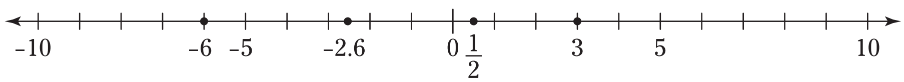


FIGURE 1-2: A number line from −10 to 10 with one-unit increments.

EXAMPLE

Q. Place the numbers $3,\ -6,\ \frac{1}{2},\ -2.6$ on a number line.

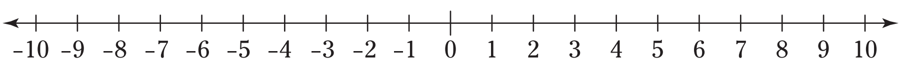
A.



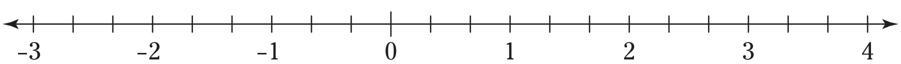
Note that the marks representing these numbers on the number line are marked with dots or points. The points for the fraction and decimal numbers are approximated, because the tickmarks for these numbers aren't on the number line to make them exact.

YOUR TURN

7 Place the following numbers on the number line: $-6,\ -1,\ 0.5,\ 2,\ 3.2$



8 Place the following numbers on the number line: $-2\frac{2}{3}, \frac{1}{3}, 2\frac{1}{3}, 3\frac{2}{3}$



Speaking in Algebra

Algebra and symbols in algebra are like a foreign language. They all mean something and can be translated back and forth as needed. It's important to know the vocabulary in a foreign language; it's just as important in algebra.

Being precise with words

The words used in algebra are very informative. You need to know their exact meaning, because they convey what is happening.

- An *expression* is any combination of values and operations that can be used to show how things belong together and compare to one another. $2x^2+4x$ is an example of an expression. Think of an expression as being the equivalent of a phrase or part of a sentence; you have some subjects and conjugates, but no verbs. You see how items are distributed over expressions in Chapter 9.
- A *term,* such as $4xy$, is a grouping together of one or more *factors* (variables and/or numbers) all connected by multiplication or division. In this case, multiplication is the only thing connecting the number with the variables. Addition and subtraction, on the other hand, separate terms from one another. For example, the expression $3xy+5x-6$ has three *terms.*
- An *equation* uses a sign to show a relationship — that two things are equal. By using an equation, tough problems can be reduced to easier problems and simpler answers. An example of an equation is $2x^2+4x=7$. See Chapters 14 through 18 for more information on equations.
- An *operation* is an action performed upon one or two numbers to produce a resulting number. Operations include addition, subtraction, multiplication, division, square roots, and so on. See Chapter 7 for more on operations.
- A *variable* is a letter representing some unknown; a variable always represents a number, but it *varies* until it's written in an equation or inequality. (An *inequality* is a comparison of two values. For more on inequalities, turn to Chapter 19.) Then the fate of the variable is set — it can be solved for, and its value becomes the solution of the equation. By convention, mathematicians usually assign letters at the end of the alphabet to be variables to be solved for in a problem (such as x, y, and z).
- A *constant* is a value or number that never changes in an equation — it's constantly the same. The number 5 is a constant because it is what it is. A letter can represent a constant if it is assigned a definite value. Usually, a letter representing a constant is one of the first letters in the alphabet. In the equation $ax^2+bx+c=0$, c is a constant and x is the variable.
- A *coefficient* is another type of constant. It is a multiplier of a variable. In the equation $ax^2+bx+c=0$, a and b are coefficients. They have constant, assigned values and are factors, but they have the special role of multiplying variables.
- An *exponent* is a small number written slightly above and to the right of a variable or number, such as the 2 in the expression 3^2. It's used to show repeated multiplication. An exponent is also called the *power* of the value. For more on exponents, see Chapter 5.

EXAMPLE

Q. Identify the terms, coefficients, factors, exponents, and constants in the expression $4x^2 - 3x + 2$.

A. There are three terms, separated by the subtraction and addition symbols. In the first term, the 4 is the coefficient, and the 4 and x^2 are factors. The 2 is the exponent. In the second term, the 3 and the x are factors. The exponent 1 isn't shown on the x; it's just assumed. And the final term, the 2, is a constant.

Q. Identify the terms, coefficients, factors, exponents, and constants in the expression $P\left(1+\frac{r}{n}\right)^{nt}$.

A. This expression has just one term. The P is a factor, and the parentheses form the other factor. There are two terms in the parentheses, and the exponent on the parentheses is nt.

YOUR TURN

9 How many terms are there in the expression: $4x - 3x^3 + 11$?

10 How many factors are found in the expression: $3xy + 2z$?

11 Which are the variables and which are the constants in the expression: $\frac{(x-h)^2}{a} + \frac{(y-k)^2}{b} = 1$?

12 Which are the exponents in the expression: $z^2 + z^{1/2} - z$?

Describing the size of an expression

An expression is a combination of terms and operations and can take on many different formats. In Chapters 11 through 16, you see many types of expressions and several types of equations that are created from the expressions. Many of these expressions and equations have very precise and descriptive names.

- A *polynomial* is an expression containing variables and constants. It consists of one or more terms. The terms are separated by addition and subtraction. And the exponents on the variable terms are always whole numbers, never fractions or negative numbers.

  Ex: $5x^4 - 2x^3 + x - 13$
- A *monomial* is a polynomial consisting of exactly 1 term.

  Ex: $15y$
- A *binomial* is a polynomial consisting of exactly 2 terms.

  Ex: $x^2 - 1$
- A *trinomial* is a polynomial consisting of exactly 3 terms.

  Ex: $3x^2 - 4x + 15$
- A *linear expression* is a polynomial in which there is no variable with an exponent greater than 1. In fact, the exponents can be only 1 or 0. And there must be at least one variable term with an exponent of 1.

  Ex: $5y - 3$
- A *quadratic expression* is a polynomial in which there is no variable with an exponent greater than 2. In fact, the exponents can be only 2, 1, or 0. And there must be at least one variable term with an exponent of 2.

  Ex. $\frac{1}{2}x^2 - 13x$

Relating operations with symbols

The basics of algebra involve symbols. Algebra uses symbols for quantities, operations, relations, or grouping. The symbols are shorthand and are much more efficient than writing out the words or meanings. But you need to know what the symbols represent, and the following list shares some of that information. The operations are covered thoroughly in Chapter 6.

- **+** means *add* or *find the sum* or *more than* or *increased by;* the result of addition is the *sum.* It also is used to indicate a *positive number.*
- – means *subtract* or *minus* or *decreased by* or *less;* the result is the *difference.* It's also used to indicate a *negative number.*
- $\times$ means *multiply* or *times.* The values being multiplied together are the *multipliers* or *factors;* the result is the *product.* Some other symbols meaning *multiply* can be grouping symbols: ( ), [ ], { }, ·, *. In algebra, the $\times$ symbol is used infrequently because it can be confused with

the variable *x.* The $\cdot$ symbol is popular because it's easy to write. The grouping symbols are used when you need to contain many terms or a messy expression. By themselves, the grouping symbols don't mean to multiply, but if you put a value in front of or behind a grouping symbol, it means to multiply.

- $\div$ means *divide*. The number that's going into the *dividend* is the *divisor.* The result is the *quotient.* Other signs that indicate division are the fraction line and slash, /.
- $\sqrt{\ }$ means to take the *square root* of something — to find the number, which, multiplied by itself, gives you the number under the sign. (See Chapter 6 for more on square roots.)
- $|\ |$ means to find the *absolute value* of a number, which is the number itself or its distance from 0 on the number line. (For more on absolute value, turn to Chapter 2.)
- π is the Greek letter pi that refers to the irrational number: 3.14159. It represents the relationship between the diameter and circumference of a circle.
- $[\ ]$ is the *greatest integer* operation. It tells you to evaluate what's in the brackets and replace it with the biggest integer that is not larger than what's in them.

EXAMPLE

Q. Use mathematical symbols to write the expression: "The product of 6 and *a* is divided by the sum of the square of *a* and 1 and added to the square root of the difference between pi and *r* cubed."

A. $\frac{6a}{a^2+1}+\sqrt{\pi-r^3}$. You don't need a dot between the 6 and the *a*. Writing the two factors together indicates a product. Putting the binomial a^2+1 in the denominator indicates that you're dividing the 6*a* by that expression. The exponent of 3 indicates *r* is being cubed. The two terms in the difference both appear under the radical.

Q. Use mathematical symbols to write the expression: "The absolute value of the sum of *x* and 8 times the greatest integer value of the quotient of *x* and 3."

A. $|x+8|\cdot\left[\frac{x}{3}\right]$. The dot between the absolute value and greatest integer operations isn't really necessary, but it helps define better what you're expressing.

YOUR TURN

13 Use mathematical symbols to write the expression: "Four times *z* plus the square root of 11."

14 Use mathematical symbols to write the expression: "The difference between *x* and two is divided by pi."

15 Use mathematical symbols to write the expression: "The product of six and the absolute value of the sum of eight and y is divided by the square root of the difference between 9 and x."

Taking Aim at Algebra-Speak

Everything you study requires some understanding of the vocabulary and any special notation. When you can use one word like "introductory" instead of "all that good stuff that comes before the meat of the matter," then you've saved time and space and gotten to the point quickly.

Algebra is full of good words and symbols, as you see in the previous section. And now you find how specific symbols and wording gets right to the point (and, yes, a point in algebra can mean multiply). You're "equal" to the challenge!

Herding numbers with grouping symbols

Before a car manufacturer puts together a car, several different things have to be done first. The engine experts have to construct the engine with all its parts. The body of the car then has to be mounted onto the chassis and also secured. Other car assemblers have to perform the tasks that they specialize in as well. When these tasks are all accomplished in order, the car can be put together. The same is true with algebra. You have to do what's inside the *grouping* symbol before you can use the result in the rest of the equation.

Grouping symbols tell you that you have to deal with the *terms* inside the grouping symbols *before* you deal with the larger problem. If the problem contains grouped items, do what's inside a grouping symbol first, and then follow the order of operations. The grouping symbols are as follows.

- **Parentheses ( ):** Parentheses are the most commonly used symbols for grouping.
- **Brackets [ ] and braces { }:** Brackets and braces are also used frequently for grouping and have the same effect as parentheses. Using the different types of symbols helps when there's more than one grouping in a problem. It's easier to tell where a group starts and ends.

- **Radical** $\sqrt{\ }$ **:** This is used for finding roots.
- **Fraction line (called the *vinculum*):** The fraction line also acts as a grouping symbol — everything above the line (in the *numerator*) is grouped together, and everything below the line (in the *denominator*) is grouped together.

Even though the order of operations and grouping-symbol rules are fairly straightforward, it's hard to describe, in words, all the situations that can come up in these problems. The explanations and examples in Chapters 3 and 7 should clear up any questions you may have.

EXAMPLE

Q. What are the operations found in the expression: $3y - \frac{y}{4} + \sqrt{2y}$?

A. The operations, in order from left to right, are multiplication, subtraction, division, addition, multiplication, and square root. The term 3*y* means to multiply 3 times *y*. The subtraction symbol separates the first and second terms. Writing *y* over 4 in a fraction means to divide. Then that term has the radical added to it. The 2 and *y* are multiplied under the radical, and then the square root is taken.

Q. Identify the grouping symbols shown in $\frac{4[6-2(x+1)]}{x^2+11}$.

A. The first grouping symbol to recognize is the fraction line. It separates the term in the numerator, $4[6-2(x+1)]$, from the terms in the denominator, x^2+11. Next, you see the brackets, which contain the two terms forming a subtraction, $[6-2(x+1)]$. The last grouping symbols are the parentheses, which have a multiplier of 2 and, inside, the sum of two terms.

YOUR TURN

16 Write the expression using the correct symbols: "The square root of *x* is subtracted from 3 times *y*."

17 Write the expression using the correct symbols: "Add 2 and *y*; then divide that sum by 11."

18 Identify the grouping symbols in $5\left\{16+\frac{4(11-z)}{12}\right\}$.

Defining relationships

Algebra is all about relationships — not the he-loves-me-he-loves-me-not kind of relationship, but the relationships between numbers or among the terms of an expression. Although algebraic relationships can be just as complicated as romantic ones, you have a better chance of understanding an algebraic relationship. The symbols for the relationships are given here. The equations are found in Chapters 14 through 18, and inequalities are found in Chapter 19.

- **=** means that the first value *is equal to* or the same as the value that follows.
- $\neq$ means that the first value *is not equal to* the value that follows.
- $\approx$ means that one value is *approximately the same* or *about the same* as the value that follows; this is used when rounding numbers.
- $\leq$ means that the first value is *less than or equal to* the value that follows.
- **<** means that the first value is *less than* the value that follows.
- $\geq$ means that the first value is *greater than or equal to* the value that follows.
- **>** means that the first value is *greater than* the value that follows.

EXAMPLE

Q. Write this expression using mathematical symbols: "When you square the sum of *x* and 4, the result is greater than or equal to 23."

A. $(x+4)^2 \geq 23$. The point of the inequality symbol always faces the smaller value.

Q. Write this expression using mathematical symbols: "The circumference, C, of a circle divided by the diameter, *d*, is equal to pi, which is about 3.1416."

A. $\frac{C}{d} = \pi \approx 3.1416$. The wavy equal sign means "approximately" or "about."

YOUR TURN

Write the expression using the correct symbols.

19 When you multiply the difference between *z* and 3 by 9, the product is equal to 13.

20 Dividing 12 by *x* is approximately the cube of 4.

21 The sum of y and 6 is less than the product of x and -2.

22 The square of m is greater than or equal to the square root of n.

Taking on algebraic tasks

Algebra involves symbols, such as variables and operation signs, which are the tools that you can use to make algebraic expressions more usable and readable. These things go hand in hand with simplifying, factoring, and solving problems, which are easier to solve if broken down into basic parts. Using symbols is actually much easier than wading through a bunch of words.

- To *simplify* means to combine all that can be combined, using allowable operations, to cut down on the number of terms, and to put an expression in an easily understandable form.
- To *factor* means to change two or more terms to just one term using multiplication. (See Chapters 11 through 13 for more on factoring.)
- To *solve* means to find the answer. In algebra, it means to figure out what the variable stands for. You solve for the variable to create a statement that is true. (You see solving equations and inequalities in Chapters 14 through 19.)
- To *check* your answer means to replace the variable with the number or numbers you have found when solving an equation or inequality, and show that the statement is true.

EXAMPLE

Q. *Simplify* the expression: $4+9-x$

A. You can add the two numbers. Simplified, you get $13-x$. You see all the types of simplifying methods in Chapter 7.

Q. *Factor* the expression: $5\cdot 6+5\cdot 11$

A. The two terms have a common factor of 5. Make this one term by taking out the common factor and writing it times the sum of the remaining factors. This can be $5(6+11)$ or, simplifying, it's $5(17)=85$. Factoring methods are found in Chapters 11 through 13.

Q. *Solve* the equation for the value of x: $x - 3 = 0$.

A. The only number that will make this equation a true statement is the number 3. So $x = 3$. Solving equations is covered in Chapters 14 through 18.

Q. *Check* to see if it's true that $x = -2$ in the equation $5 + x = 3$.

A. Replace the x in the equation and simplify on the left. You have $5 + (-2) = 3$, which is the same as $5 - 2 = 3$ or $3 = 3$. Yes, this is the solution to the equation.

YOUR TURN

23 Simplify the expression $9 - 8 + 2x + 7x$.

24 Factor the expression $14y - 28z$.

25 Solve for the value of z in the equation $8 + z = 10$.

26 Check to see if both –3 and 3 are solutions to the equation $x^2 = 9$.

Practice Questions Answers and Explanations

1. **The numbers 15, 11, and $\sqrt{9}$ are natural numbers.** The number $\sqrt{9}$ qualifies, because it simplifies to the number 3.

2. **The numbers −41, 15, 11, $-\frac{12}{3}$, and $\sqrt{9}$ are integers.** The fraction $-\frac{12}{3}$ simplifies to −4, and $\sqrt{9}$ is equal to 3.

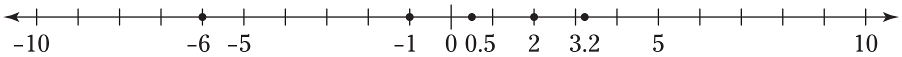
3. **The numbers $-41,\ 15,\ -5.2,\ 11,\ 3.2121...,\ -\frac{12}{3},\ \frac{14}{11}$, and $\sqrt{9}$ are all rational numbers.**

   They can all be written as a fraction with integers in the numerator and denominator.

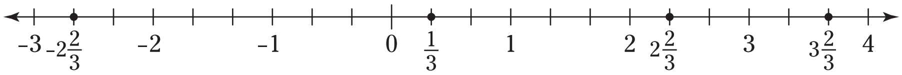
4. **The number $\sqrt{10}$ is the only irrational number listed here.** Technically, $\sqrt{-5}$ can be written as $\sqrt{5}i$, but the i makes the number imaginary. And the $\sqrt{9}$ can be simplified to create a rational number.

5. **The numbers 11 and 3 are prime.** Yes, $\sqrt{9}$ appears again!

6. **The only imaginary number is $\sqrt{-5}$ or $\sqrt{5}i$.**

7. 

   The points 0.5 and 3.2 are approximated, because the tick marks aren't on the number line.

8. 

9. **3.** There are three terms. The term $4x$ is separated from $3x^3$ by subtraction and from 11 by addition.

10. **5.** There are two terms, and each has a different number of factors. The first term, $3xy$, has three factors: the 3 and the x and the y are multiplied together. The second term, $2z$, has two factors, the 2 and the z. So there are a total of five factors.

11. **x and y; h, k, a, b, 1.** The two variables are x and y. The constants are h, k, a, b, and 1.

12. **2, $\frac{1}{2}$, and 1.** The exponent in the term z^2 is the 2; the z is the base. The exponent in $z^{1/2}$ is $\frac{1}{2}$. And, even though it isn't showing, there's an implied exponent in the term z; it's assumed to be a 1, and the term can be written as z^1.

13. $4z+\sqrt{11}$**.** You could put the dot between the 4 and the z to indicate multiplication, but writing them together assumes they're being multiplied.

14. $\frac{x-2}{\pi}$**.** Use a fraction to indicate multiplication, rather than the slash.

15. $\frac{6|8+y|}{\sqrt{9-x}}$**.** Writing the 6 next to the absolute value indicates multiplication. The subtraction is written completely under the radical.

16. $3y-\sqrt{x}$**.**

17. $\frac{2+y}{11}$ **or** $(2+y)/11$.

18 The braces contain two terms: the 16 and the fraction. The 5 in front of the braces indicates that multiplication will be performed. The fraction line has the term $4(11-z)$ in the numerator and the number 12 in the denominator. And the parentheses have a multiplier of 4 in front and the two terms 11 and z inside, which need to be subtracted.

19 $\mathbf{(z-3)9=13}$ **or** $\mathbf{9(z-3)=13}$**.** The 9 can be written behind or in front of the parentheses.

20 $\frac{12}{x}\approx 4^3$. The x goes in the denominator.

21 $\mathbf{y+6<-2x}$ **or** $\mathbf{y+6<x(-2)}$. Use parentheses if the −2 follows the x.

22 $m^2 \geq \sqrt{n}$. Use the greater-than-or-equal-to symbol.

23 $\mathbf{1+9x}$**.** You subtract the 8 from the 9, and you add the $2x$ and $7x$ to get $9x$. (It's like adding 2 apples to 7 apples!)

24 $14(y-2z)$. Both $14y$ and −28 are divisible by 14. So you write the 14 outside the parentheses and put the division results inside the parentheses.

25 $\mathbf{z=2}$**.** The only number you can add to 8 to get a result of 10 is 2.

26 **Yes, they are both solutions.** $(3)^2=9$ and $(-3)^2=9$

If you're ready to test your skills a bit more, take the following chapter quiz that incorporates all the chapter topics.

Whaddya Know? Chapter 1 Quiz

Complete each problem. You can find the solutions and explanations in the next section.

1 Which of the following numbers can be termed *rational*?

$-3,\ \frac{13}{11},\ \sqrt{-25},\ 4.431321,\ 102$

2 Which of the following is a solution of the equation $\frac{1+x}{5}=3$?

A) 5 B) −4 C) 14 D) 15 E) 16

3 How many terms are there in $2x(x-4)$?

4 What is the exponent in $\frac{(x-h)^2}{3}-\frac{(y-k)^2}{4}=1$?

5 Which of the following is: "the product of four times the difference between x and eight divided by the sum of nine and x^2"?

A) $\frac{4x-8}{9+x^2}$ B) $\frac{4(x-8)}{9+x^2}$ C) $\frac{4(x+8)}{9+x^2}$ D) $\frac{x-8}{4(9-x^2)}$ E) $\frac{9-x^2}{4(x-8)}$

6 Write the following using the corresponding mathematical symbols: "The quotient of x and 11 is greater than or equal to the sum of 9 and y."

7 Write the following using the corresponding mathematical symbols: "The difference between 9 and z is less than the product of 9 and z."

8 How many terms are there in $3x^2 - 5x - 2$?

9 What is a common factor of the terms $3a + 3a^2 + 3a^3 + 3$?

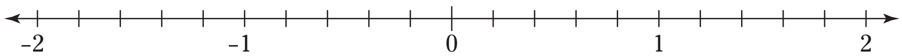
10 Which of the following numbers can be termed real?

$$-3.\overline{42},\ \frac{13}{111},\ 4+3i,\ 53^2,\ -\frac{1}{1001}$$

11 What is the constant term in $\frac{x}{4} + \frac{y}{9} = 1$?

12 What is the coefficient in $x^2 + 5x - 11$?

13 Place the following numbers on the number line: $-1.5,\ -0.6,\ 0.8$



14 Write the following using the corresponding mathematical symbols: "The square root of 20 is about 4.5."

15 Simplify: $4x^2 + 3 - x^2 + 7$.

Answers to Chapter 1 Quiz

1. $-3, \frac{13}{11}, 4.431321, 102$. All of the terms are rational except for $\sqrt{-25}$. The number -3 is an integer and is rational, because it can be written as a fraction such as $-\frac{3}{1}$. The number $\frac{13}{11}$ is already written as a fraction and is rational. The number 4.431321 is a terminating decimal and can be written as $4\frac{431321}{1{,}000{,}000}$ or $\frac{4{,}431{,}321}{1{,}000{,}000}$. The number 102 is a whole number and can be written as a fraction over 1. The number $\sqrt{-25}$ is imaginary, so it can't be a rational number.

2. **C. 14.** Replacing the x with 5, you get $\frac{6}{5}$, which is not equal to 3. Replacing the x with -4, you get $\frac{-3}{5}$, which is not equal to 3. Replacing the x with 15, you get $\frac{16}{5}$, which is not equal to 3. Replacing the x with 16, you get $\frac{17}{5}$, which is not equal to 3. Replacing the x with 14, you get $\frac{15}{5}$, which IS equal to 3.

3. **1**. The $2x$ multiplies a binomial, making the expression all one term. There are two terms in the parentheses, but the expression is still just one term.

4. **2**. There are two exponents, each on the terms in the parentheses. They are both 2.

5. **B.** The answer $\frac{4x-8}{9+x^2}$ only multiplies the x and not the 8. The answer $\frac{4(x+8)}{9+x^2}$ finds the sum of x and 8, not the difference. The answer $\frac{x-8}{4(9-x^2)}$ has the 4 multiplier in the denominator instead of the numerator. And the answer $\frac{9-x^2}{4(x-8)}$ has reversed the numerator and denominator.

6. $\frac{x}{11} \geq 9+y$. The quotient refers to division, and the sum refers to addition. The point of the symbol faces the smaller side.

7. $9-z<9z$. The difference refers to subtraction, and the product refers to multiplication. The point of the symbol faces the smaller result.

8. **3**. The three terms are separated by the two subtraction symbols.

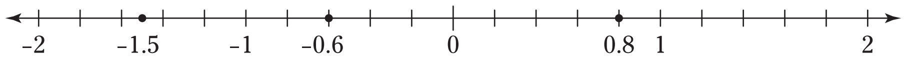
9. **3**. The number 3 divides each of the terms evenly (leaving no remainder).

10. $-3.\overline{42}, \frac{13}{111}, 53^2, -\frac{1}{1001}$ All of the terms are real except for $4+3i$. The number $-3.\overline{42}$ is a repeating decimal and is a rational number; it can be written as $-3\frac{14}{33}$ or $-\frac{113}{33}$. See Chapter 4 for more on repeating decimals. The number $\frac{13}{111}$ is a rational number and so is real. The number 53^2 is equal to 2,809 and is a whole number. And $-\frac{1}{1001}$ is a rational number, already written as a fraction. The number $4+3i$ has the imaginary factor of i, so it is imaginary and not real.

11 **1**. The number 1 is a term that stands alone and isn't multiplying or dividing any other number. The 4 and 9 are both part of the coefficients of their respective terms.

12 **5.** The 5 multiplies the variable x.

13



The number line is broken up into units of 0.2 in length. The number –1.5 has to be estimated, as it's halfway between –1.6 and –1.4. The other two numbers have tick marks to rest on.

14 $\sqrt{20} \approx 4.5$. The "wavy equal sign" symbol means the answer is approximate and has been rounded.

15 $3x^2 + 10$. The two x^2 terms are combined by subtracting 1 from 4. The two constants are added together.